V_{avg} = average fluid velocity in a duct, cm./sec.

$V_{0,0}, V_{2,-1}$, etc. = values of V at the mesh nodes indicated in Figure 3, dimensionless

$\overline{V}$ = computed quantity used in Equations (15) through (19), dimensionless

W = overrelaxation factor, dimensionless

x', y', z' = rectangular Cartesian coordinates, cm.

x, y, z = dimensionless coordinates, such as $x = x'/L$

x_0, y_{-1}, etc. = values of x and y at particular mesh nodes as indicated in Figure 3, dimensionless

∇^2 = $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}$, dimensionless

τ_{max} = maximum shear stress exerted on the fluid in a given experiment, dynes/(sq. cm.)

ρ = fluid density, g./cc.

μ = $m \left[\left(\frac{\partial v'}{\partial x'} \right)^2 + \left(\frac{\partial v'}{\partial y'} \right)^2 \right]^{n-1/2}$, local viscosity at a point in the square duct, dyne sec./(sq. cm.)

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Part II. Experimental Results

The friction factor and Reynolds number were measured for sodium carboxymethylcellulose (cmc) flowing through a rectangular duct. Measured values were compared with values calculated in Part I of this paper. Both a pipe viscometer and a Couette viscometer were used to evaluate the rheological parameters for the fluids. Calculations were based on a power law model. The agreement between theoretical and experimental values was excellent.

Two papers have recently been presented discussing the laminar flow of pseudoplastic fluids through long rectangular ducts. Schechter (3) devised a variational principle which he used to obtain approximate solutions for the equation of motion, while Wheeler and Wissler (5) used an iterative finite-difference method. In both papers, velocity profiles and the friction factor-Reynolds number product were calculated for several rectangular cross sections and a range of fluid properties.

One can anticipate that experimentally obtained data might differ significantly from the corresponding calculated data for one or more of the following reasons: the rheological model is inadequate to describe the fluid, the approximate solutions are not sufficiently accurate, or experimental errors are too large. Clearly, computational and experimental errors must be kept small if one hopes to draw any conclusions about the adequacy of the model.

The principal reason for suspecting that the rheological model might be inadequate is that many pseudoplastic fluids are also viscoelastic. Consequently, normal stresses, as well as the usual shear stresses, are developed when the fluid is pumped through a square duct, and this can lead to a transverse secondary flow which is superimposed on the main axial flow. The transverse secondary flow is manifest in a modified main flow due to both changes in the shear stresses and the existence of nonzero acceleration in the equation of motion.

In this paper the authors are presenting some reasonably accurate experimental data which can be used to check the validity of neglecting secondary flow, at least as a first approximation. The data were consolidated into pairs of dimensionless numbers consisting of the friction factor and the Reynolds number. Admittedly, the friction factor-Reynolds number correlation is not as sensitive to details of the rheological model as are the velocity profiles because the Reynolds number depends only on the average velocity. It is possible that small but theoretically significant deviations from the predicted velocity profiles are not reflected to a detectable extent in the observed correlation. However, large differences between the true velocity profile and the predicted profile should be detectable in the friction factor correlation.

Some idea of the inaccuracy of the computed results can be obtained by comparing corresponding results obtained by two independent workers. Although completely

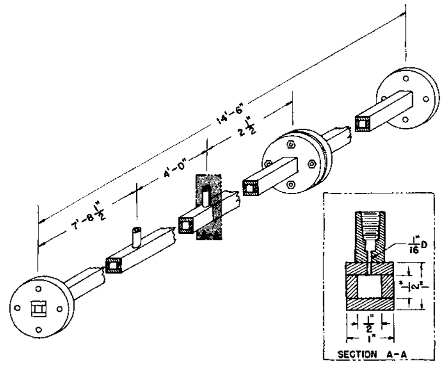

Fig. 1. Duct test section.

TABLE 1. POWER LAW MODEL PARAMETERS

Run no.	Weight % CMC	Pipe n	Pipe m $\frac{\text{dyne-sec.}^n}{\text{sq. cm.}}$	Pipe Stress range $\frac{\text{dynes}}{\text{sq. cm.}}$	Viscometer n	Viscometer m $\frac{\text{dyne-sec.}^n}{\text{sq. cm.}}$	Viscometer Stress range $\frac{\text{dynes}}{\text{sq. cm.}}$	Square duct Stress range* $\frac{\text{dynes}}{\text{sq. cm.}}$
VII	1.0	0.475	21.8	420-1,200	0.515	17.5	250-640	563-1,040
VIII	1.1	0.437	36.3	570-1,400	0.484	27.1	330-800	691-1,310
IX	0.51	0.641	2.55	120-460	0.680	2.04	67-240	163-424
X	0.66	0.564	6.25	360-700	0.619	4.49	110-340	323-734
XI	0.90	0.526	12.7	440-1,200	0.543	12.0	200-540	416-1,140
XII	0.52	0.611	3.25	220-450	0.670	2.25	70-240	178-541
XIII	0.99	0.487	19.8	590-1,500	0.517	16.7	240-620	563-1,270
XIV	0.32	0.766	0.669	67-240	0.727	0.880	37-140	49-227
XV	0.51	0.592	3.86	170-480	0.663	2.38	71-240	177-446
XVI	0.28	0.731	0.721	49-160	0.737	0.708	30-120	59-193

* Calculated from power law parameters determined from pipe data.

different computational schemes were used in the two papers mentioned above, the agreement between calculated results was quite good; differences varied from 0.3 to 1.0%. In addition, evidence was presented by Wheeler and Wissler to support their claim that computational errors had been reduced to no more than 0.1%. These should be tolerable errors.

EQUIPMENT AND EXPERIMENTAL PROCEDURE

A closed system was used to obtain pressure drop data for the flow of CMC solutions through a duct of square cross section and a pipe. The duct test section was constructed of brass bar stock; its essential dimensions are given in Figure 1. Micrometer measurements taken between the two pressure taps established that the inside width of the duct was 0.502 ± 0.004 in., and the depth was 0.500 in. In view of the relatively small error involved, computations were based on a square cross section of exactly 0.5 in. on a side. The pipe test section was constructed from ⅝ in. copper tube with an inside diameter of 0.460 in. For both the pipe and the duct, care was taken to butt the channel components together such that the bore surfaces were aligned, and the inside surfaces were polished to near mirror smoothness. It was assumed that the inlet region of 180 equivalent pipe diameters preceding the first pressure tap was sufficient to render inlet effects negligible.

Two glass U tubes were used for manometers. One tube contained mercury, the other water saturated tetrabromoethane. The ¼ in. copper tubing lead lines were filled with CMC solution. The manometers were read by means of a cathetometer, and the manometer temperatures were observed with a mercury thermometer.

A magnetic flow meter was used to measure flow rates, and a permanent record of the flow rates was made by a recorder. The flow meter was calibrated directly for each CMC solution by determining the time required for a known volume to be accumulated in a holdup tank. Recorded flow rates were believed to be in error by less than 3%.

Temperature regulation was accomplished by a 10-ft. coil of ¼ in. copper tubing enclosed in a 2-in. I.D. brass pipe connected to the pump discharge. Chilled water was circulated through the coil to remove the heat generated by viscous dissipation in the system. The temperature regulation system sufficed to hold the temperature of the test section effluent within ± 0.1°C. of 35°C.

Stress-strain rate data were obtained with a concentric cylinder viscometer which was modified by the addition of a cooling coil and a thermometer. In this instrument the stationary inner cylinder is connected to a torque gauge, and the outer cylinder can be rotated at speeds of 6, 100, 200, 300, and 600 rev./min. The torque gauge was calibrated by connecting the inner cylinder to an analytical balance by means of a pulley system. The torque gauge reading D and the stress $\tau_{r\theta}|_{r=R_1}$ on the inner cylinder measured in dynes per square centimeters were found to be related by the equation

$$\tau_{r\theta}|_{r=R_1} = 5.64\,(D - 0.00031\,D^2) \tag{1}$$

Nominal speeds of the outer cylinder were checked by a stroboscope. No difference between the stroboscope settings and the nominal speeds was detected.

Approximately 100 liters of CMC solution were prepared for each of the ten test runs. Hercules Powder Company Cellulose, Gum, code 51207124111, (sodium carboxymethylcellulose) in powdered form was dissolved in water to produce the test solutions. No special effort was made to obtain accurately known compositions. The concentrations in Table 1 are accurate to within ± 5.0% of the listed values. A quantity of mercuriphenyl nitrate equal to about 1.0% by weight of the quantity of CMC was added to each solution to prevent deterioration by biological action. The CMC solutions were stirred vigorously for about 8 hr. to insure homogeneity. Entrained air was removed from the solution by allowing it to stand in the mixing tanks and by repeated circulation through a centrifugal separator.

After entrained air had been removed, the test solution was circulated through the pipe test section at the highest flow rate of interest. The flow meter discharge was returned directly to the circulating pump intake rather than being routed through the mixing tanks. When steady state had been established, readings of the flow rate recorder and the manometer were made. The term *steady state* means that the flow rate was constant and the temperature of the test section discharge remained sationary to within ± 0.1°C. of 35.0°C. for at least 3 min. The process was repeated for successively smaller flow rates until the flow recorder indicated about 10% of full-scale deflection. When the measurements on the pipe test section were completed, the test solution was routed through the square duct test section.

Torque gauge readings were obtained at four speeds, 100, 200, 300, and 600 rev./min., for each of two samples. After a torque measurement had been made at each of the four speeds, the same set of measurements was repeated. Usually the two sets of measurements agreed within ± 0.2 gauge divisions. If this were not the case, a third set of measurements was made on the sample.

DATA CORRELATION

From the method of least squares, two sets of the power law parameters, m and n, were determined for each CMC solution, one from the pipe data and the other from the viscometer data.

The analysis of the pipe data was based on the well-known equation (*1*)

$$\frac{dP}{dz} = 2m\,\frac{V^n_{\text{avg}}}{R^{n+1}}\left(\frac{1}{n} + 3\right)^n \tag{2}$$

Reynolds number and friction factor values were computed for each pipe datum point by the definitions for the power law model:

$$N_{ReP} = \frac{2R^n \rho V^{2-n}_{avg}}{m} \quad (3)$$

$$f_P = \frac{-R}{\rho V^2_{avg}} \frac{dP}{dz} \quad (4)$$

The form of the regression curve was taken to be

$$f'_P = \frac{4}{N_{ReP}} \left(\frac{1}{n} + 3 \right)^n \quad (5)$$

which is equivalent to Equation (2). Data points for which the Reynolds number exceeded 2, 100 were excluded from the parameter determination. The computed values of m and n for each of the ten solutions are contained in Table 1. All of the original data are presented elsewhere (4).

A per cent standard deviation σ_P which provides a measure of the lack of agreement between the predicted and observed friction factors was calculated by the formula

$$\sigma_P^2 = \frac{10^4}{\text{d.f.}} \sum_i \left(\frac{f_{Pi} - f'_{Pi}}{f_{Pi}} \right)^2 \quad (6)$$

where i = index running over all pipe data points for which $N_{ReP} < 2{,}100$, and d.f. = degrees of freedom. Both inadequacy in the rheological model and erperimental error contribute to σ_P. The number of degrees of freedom, 51, was obtained by subtracting the number of parameters, 20, determined from the pipe data from the number of data points, 71, included in the analysis. The percent standard deviation was found to be 2.5.

The stress ranges for which the power law parameters are applicable were determined by computing the pipe wall stress for the largest and smallest flow rates for each of the ten runs by the formula

$$\tau_{max} = \frac{mV^n_{avg}}{R^n} \frac{f_P N_{ReP}}{4} \quad (7)$$

The stress ranges are included with the power law parameters in Table 1.

The analysis of the viscometer data was based on the following equation which relates the torque $\tau_{r\theta}$ exerted on the inner cylinder to the angular velocity Ω of the outer cylinder:

$$\tau_{r\theta}\Big|_{r=R_1} = m \left\{ \frac{2\Omega}{n\{1 - (R_1/R_2)^{2/n}\}} \right\}^n \quad (8)$$

Equation (1) was used to compute the observed stress on the inner cylinder from the torque gauge reading. Values for two dimensionless numbers τ_V and τ_V' were computed for each angular velocity from the following equations:

$$\tau_V = \frac{5.640}{\rho R_2^2 \Omega^2} (D - 3.1 \times 10^{-4} D^2) \quad (9)$$

$$\tau_V' = \frac{m}{\rho R_2^2 \Omega^2} \left\{ \frac{2\Omega}{n[1 - (R_1/R_2)^{2/n}]} \right\}^n \quad (10)$$

The number τ_V is independent of the estimated values of m and n and directly proportional to the observed stress. The number τ_V' is a function of the parameters n and m and not directly a function of the observed stress.

The reduced standard deviation for the empirical regression curve is defined by

$$\sigma_V^2 = \frac{10^4}{\text{d.f.}} \sum_i \left(\frac{\tau_{Vi} - \tau'_{Vi}}{\tau_{Vi}} \right)^2 \quad (11)$$

The number of degrees of freedom was 20, found by subtracting the number of parameters determined, 20, from the number of data points, 40. The sum in Equation (11) included all of the viscometer data points for all ten CMC solutions, and it was found that $\sigma_V = 3.7\%$.

The two independent pairs of power law parameters determined for each of the CMC solutions were used separately to correlate the duct data. Since the method of application was identical for the two sets, only the evaluation based on the pipe parameters will be considered in detail.

A Reynolds number for each of the flow rates observed in the duct was computed from the pipe parameters tabulated in Table 1:

$$N_{ReD} = \frac{L^n \rho V^{2-n}_{avg}}{m} \quad (12)$$

The friction factors actually observed in the duct were calculated by the definition

$$f_D = \frac{-2L}{\rho V^2_{avg}} \frac{dP}{dz} \quad (13)$$

while the numerical solution

$$f_D' = \frac{7.4942}{N_{ReD}} \left(\frac{1.7330}{n} + 5.8606 \right)^n \quad (14)$$

to the equation of motion in the duct (5) was employed to predict friction factors for each N_{ReD}.

A criterion for distinguishing turbulent flow from laminar flow in the duct was required to properly evaluate the duct data. It was assumed that there exists a critical Reynolds number above which flow in the duct is turbulent. The critical value of N_{ReD} was estimated by examining the data for a sharp increase in the deviation between the observed and predicted friction factors. As the selected data in Table 2 suggest, the critical Reynolds number for the duct is about 1,500. In view of the sparsity of the experimental data close to 1,500, it can be stated with confidence only that the critical value lies between 1,400 and 1,600.

The maximum stress in the duct for each observed flow rate was obtained by interpolation from Table 3 of Part I. The values are included in Table 1.

The per cent standard deviation σ_{DP} between the observed friction factors and the friction factors predicted from pipe parameters was computed by the formula

$$\sigma^2_{DP} = \frac{10^4}{\text{d.f.}} \sum_i \left(\frac{f_{Di} - f'_{Di}}{f_{Di}} \right)^2 \quad (15)$$

Table 2. Selected Data in the Vicinity of the Critical Reynolds Number

Type of flow	Run	Data point	N_{ReD}	$\frac{f_D - f'_D}{f_D} \cdot 100$
Turbulent	XII	1	1,808	14.9
($N_{ReD} > 1{,}500$)	XII	2	1,595	12.2
	XIV	1	1,560	3.6
	XVI	1	1,758	23.3
	XVI	2	1,504	9.1
	Aver.*			14.2
Laminar	IX	1	1,161	1.2
($N_{ReD} < 1{,}500$)	X	1	1,296	2.9
	XII	3	1,360	4.9
	XIV	2	1,331	−1.9
	XVI	3	1,246	0.4
	Aver.*			2.7

* Root mean square average.

The number of degrees of freedom is equal to the number of data points, 78, included in the sum. Data points for which N_{ReD} exceeded 1,500 were excluded from the sum. The standard deviation was found to be 2.7%.

Graphical presentation of the duct data in consolidated form is facilitated by defining a modified Reynolds number

$$N_{ReM} = N_{ReD} \frac{7.5936}{\left(\frac{1.7330}{n} + 5.8606 \right)^n} \quad (16)$$

For any pair of the parameters n and m

$$f_D' = \frac{56.908}{N_{ReM}} \quad (17)$$

Note that N_{ReM} reduces to N_{ReD} for the Newtonian case ($n = 1$). A plot of f_D and f_D' vs. N_{ReM} is presented in Figure 2.

The power law parameters determined from viscometer data were used to compute the quantities N_{ReD}, f_D, f_D', and N_{ReM} as previously defined. A measure of the difference between the observed friction factors for the duct and those predicted from the viscometer parameters was provided by the standard deviation σ_{DV}, which is similar in definition to σ_{DP} of Equation (15). The value of σ_{DV} was found to be 5.2% with 78 deg. of freedom.

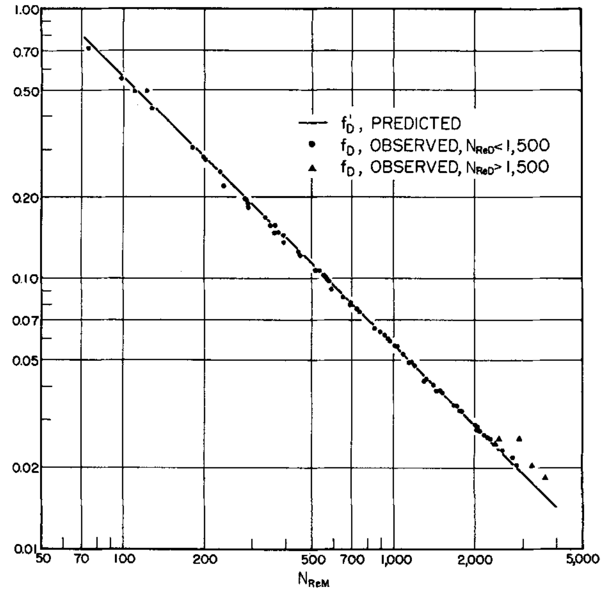

Fig. 2. Comparison of observed square duct friction factors with friction factors predicted from a stress-strain rate model based on pipe data.

DISCUSSION AND CONCLUSION

Friction factors predicted for the rectangular duct with rheological parameters based on pipe data are in good agreement with the experimentally observed friction factors. A comparison is shown in Figure 2. The agreement between the observed and predicted friction factors for the duct-viscometer correlation is slightly poorer with the measured friction factor lying consistently below the theoretical curve. Evidence indicating that the discrepancy is due to the empirical nature of the power law model will be presented in this section.

The power law model has an empirical rather than theoretical basis. In both its one-dimensional and generalized forms, it has only two adjustable parameters. The model predicts a vanishingly small viscosity at very large strain rates and an infinitely large viscosity at zero strain rate. It is evident that the ability of the model to describe the stress-strain rate relationship of a real fluid over a wide stress range is limited.

If experimental data from two stress ranges are analyzed, two pairs of power law parameters will be determined. This is apparent in Table 1. The difference between the two models is due both to experimental error and to the inadequacy of the model. The pipe and viscometer models for a typical CMC solution are shown in Figure 3. The vertical lines in the figure cut the model curves at the largest and smallest characteristic stresses for each set of data. Note that the agreement between the pipe and viscometer models is good in the region of overlapping stress ranges, but in the region of higher strain rates, shear stresses calculated from the viscometer model are larger than the corresponding pipe stresses. This was found to be the case for every one of the ten CMC solutions. Accordingly, one would expect predicted friction factors based on viscometer data to be larger than those based on pipe data, especially at the higher Reynolds numbers.

The inadequacy of the power law model is retained when the model is generalized to more than one dimension. For each of the ten runs, the stress ranges of the pipe and duct data are about the same. It can thus be assumed that the error arising from power law inadequacy is the same in the duct correlation based on parameters determined from the pipe correlation as in the pipe correlation itself. The stress ranges of the duct and viscometer data differ considerably. Generalizing a rheological model obtained from viscometer data and applying it to correlate duct data must introduce an error in the duct correlation not reflected in the viscometer correlation. This error is due not to the process of generalizing the model but rather to extrapolating a model determined for one stress range to correlate data in a different stress range.

Standard deviations were calculated for the pipe, viscometer, and duct data from Equations (6), (11), and (15), respectively. Two independent values were calculated for the duct, one based on power law parameters evaluated from pipe data and the other with viscometer data.

Each of the σ's represents a root mean square deviation between an observed stress quantity and the corresponding computed quantity. To facilitate comparison, each σ was computed on a relative, that is percentage, basis rather than an absolute basis.

It is tempting to assume that the only contribution to the σ's is a random, normally distributed experimental

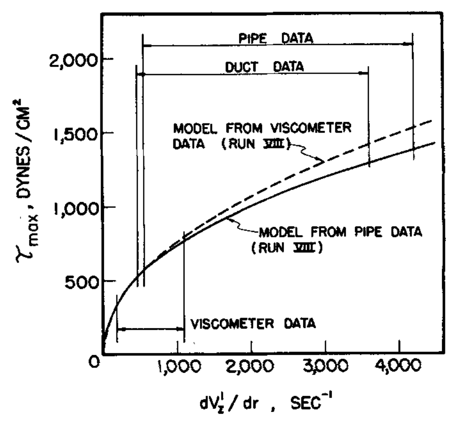

Fig. 3. Comparison of power law models based on data from different stress ranges.

error. The σ's could then be compared by standard statistical methods. Such an assumption would be invalid.

A qualitative comparison of σ_P and σ_{DP} is rather interesting. The quantity σ_P represents the average value of the error due to experimental errors in the pipe data and the inadequacy of the rheological model. The quantity σ_{DP} represents the average error due to experimental error in the duct data, inadequacy of the power law, and error arising from generalizing the one-dimensional pipe model to describe the stress-strain rate relationship in the duct. The experimental measurements on the pipe and duct systems were made with the same equipment under similar conditions. It is reasonable to assume that the magnitudes of the contributions of experimental error to σ_P and σ_{DP} are the same. As pointed out above, the effects of model inadequacy on the two averages should also be the same. Any significant difference between σ_P and σ_{DP} must then be due to some fundamental difference in the stress-strain rate relations in the pipe and duct systems. It was found that $\sigma_P = 2.5\%$ with 51 deg. of freedom and that $\sigma_{DP} = 2.7\%$ with 78 deg. of freedom. The difference is not significant. It can be safely concluded that the average error, if any, due to generalizing the model is less than 1%.

A very different situation is apparent for the viscometer-duct correlation. Neither the experimental equipment nor the stress ranges in which measurements were made are similar for the two systems. The average deviations were found to be $\sigma_V = 3.7\%$ with 20 deg of freedom and $\sigma_{DV} = 5.2\%$ with 78 deg of freedom. Furthermore, the predicted friction factors were consistently larger than those determined experimentally. In view of the comments on extrapolating the viscometer model, it is surmised that the difference between σ_V and σ_{DV} is due primarily to extrapolating the model.

The good agreement obtained in the pipe-duct correlation may be considered to be somewhat surprising in view of recent speculations about the existence of secondary flow in a square duct. It can be shown that the three-constant Oldroyd model proposed by Williams and Bird (6) predicts that normal stresses roughly equal in magnitude to τ_{xz} should exist in regions of the duct where the strain rate is large. If such large normal stresses exist, then why is there not a large discrepancy between the calculated and measured friction factor-Reynolds number relations? Perhaps the answer is that the isotropic pressure distribution in the duct is such that secondary flow is suppressed, which is certainly possible if $\tau_{xx} = \tau_{yy}$ when the flow is rectilinear. Equality of these two normal stresses is one of the properties inherent in the three-constant Oldroyd model. Another possibility is that an appreciable secondary flow does exist, and the axial velocity profile does differ significantly from the calculated profile, but the friction factor-Reynolds number correlation is unchanged. In other words, for a given axial pressure gradient the true axial velocity profile and the calculated profile differ significantly, but they both have the same average value. Visual observations made in a plastic duct have not revealed the existence of large secondary flows, which would tend to support the first hypothesis. Further experimental and theoretical work will be required before the situation is fully understood.

ACKNOWLEDGMENT

Financial support for this study was provided by grants from the University Research Institute of The University of Texas and the National Science Foundation (Research Grant NSF-G10866). It is also a pleasure to acknowledge the helpful suggestions made by Dr. R. S. Schechter during the course of this study.

NOTATION

D = torque dial reading of the concentric cylinder viscometer
f_D = observed friction factor for the square duct defined by Equation (13), dimensionless
f_D' = predicted friction factor for the square duct defined by Equation (14), dimensionless
f_P = observed friction factor for the pipe defined by Equation (4), dimensionless
f_P' = regression value of the friction factor for the pipe defined by Equation (5), dimensionless
L = width of the square duct, cm.
m = parameter in the power law model, dynes sec.^{n}/(sq. cm.)
n = parameter in the power law model, dimensionless
$(dP)/(dz)$ = axial pressure drop in a pipe or a square duct, dynes/cc.
R = pipe radius, cm.
R_1 = radius of the inner cylinder of the viscometer, cm.
R_2 = radius of the outer cylinder of the viscometer, cm.
N_{ReD} = Reynolds number for the duct calculated by Equation (12), dimensionless
N_{ReP} = Reynolds number for a pipe defined by Equation (3), dimensionless
N_{ReM} = modified Reynolds number defined by Equation (16), dimensionless
V_{avg} = average axial velocity in the pipe or duct, cm./sec.
$(dV_z')/(dr)$ = strain rate, sec.$^{-1}$

Greek Letters

σ_P = per cent standard deviation between regression and observed friction factors for the pipe defined by Equation (6)
σ_V = per cent standard deviation between τ_V and τ_V' for the viscometer defined by Equation (11)
σ_{DP} = per cent standard deviation between friction factors observed in the duct and those predicted from pipe data defined by Equation (15)
σ_{DV} = per cent standard deviation between friction factors observed in the duct and those predicted from viscometer data defined similarly to σ_{DP}
τ_V = observed stress quantity defined for the viscometer by Equation (9), dimensionless
τ_V' = regression stress quantity defined for the viscometer by Equation (10), dimensionless
τ_{max} = maximum stress in a fluid flowing at a given rate, dynes/(sq. cm.)
$\tau_{r\theta}|_{r=R_1}$ = the tangential component of the shear stress exerted by the sample on the inner cylinder of the viscometer, dynes/(sq. cm.)
Ω = angular velocity of the outer cylinder of the viscometer, sec.$^{-1}$

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Manuscript received March 23, 1964; revision received September 14, 1964; paper accepted September 16, 1964.